A Living System Must Have Noncomputable Models

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Abstract Chu and Ho's recent paper in *Artificial Life* is riddled with errors. In particular, they use a wrong definition of Robert Rosen's *mechanism*. This renders their "critical assessment" of Rosen's central proof null and void.

Keywords Robert Rosen, *Life Itself*, (M,R)-systems, noncomputability

1 Introduction

The conclusion of Chu and Ho's recent article [1] published in *Artificial Life* is that "Rosen's central proof is wrong". The "central proof" refers to the main conclusion of Robert Rosen's book *Life Itself* [4], that a living system is not a mechanism and consequently must have noncomputable models. Chu and Ho, however, use a definition of *mechanism* that is different from Rosen's. This and numerous other errors in their paper make their argument irrelevant. In this short note I shall discuss some of their errors.

To foreshadow what is to come, let me first point out that the category-theoretic definitions of product and coproduct (captions of Figures 1 and 3 in [1]) are wrong (and cannot be explained away as simple misprints). Such elementary mistakes indicate a lack of understanding of the basic concept of *universal property* in category theory.

2 The Rosen Theorems

In [4], Rosen defined the term *simulable* and several of its synonyms. A mapping is simulable if it is "definable by an algorithm." It is variously called *computable*, *effective*, and "*evaluable by a mathematical (Turing) machine*." In Chapter 8 of [4] he gave the following:

Definition 2.1 A natural system *N* is a *mechanism* if and only if all of its models are simulable.

He then proved five propositions for a mechanism *N*. In particular, "Conclusion 4" is the following:

Theorem 2.2 *Analytic and synthetic models coincide in the category* ***C***(*N*) *of all models of N; direct sum = direct product.*

And "Conclusion 5" is the following:

Theorem 2.3 *Every property of N is fractionable*.

Immediately following this, in Chapter 9 of [4], Rosen, using these five just-proven properties, presented a detailed *reductio ad absurdum* argument that *proves* that certain modes of entailment are *not* available in a mechanism:

Theorem 2.4 *There can be no closed path of efficient causation in a mechanism.*

The contrapositive statement of Theorem 2.4 is

Theorem 2.5 *If a closed path of efficient causation exists in a natural system N, then N cannot be a mechanism.*

Taking Definition 2.1 of mechanism into account, this is equivalent to

Theorem 2.6 *If a closed path of efficient causation exists for a natural system N, then it has a model that is not simulable.*

An iteration of "efficient cause of efficient cause" is inherently hierarchical. A closed path of efficient causation must form a hierarchical cycle. *Both* the hierarchy and the cycle (closed loop) are essential attributes of this closure. In formal systems, hierarchical cycles are manifested by impredicativities, or the inability to replace these self-referential loops with finite syntactic algorithms. The nonsimulable model in Theorem 2.6 contains a hierarchical closed loop that corresponds to the closed path of efficient causation in the natural system being modeled. In other words, it is a formal system with an impredicative loop of inferential entailment. Thus we also have:

Theorem 2.7 *If an impredicative loop of inferential entailment exists for a formal system, then it is not simulable.*

A natural system that has a nonsimulable model is defined by Rosen as a *complex system* (Chapter 19 of [5]). A necessary condition for a natural system to be an *organism* is that it is closed to efficient causation (Chapter 1 of [5]). Theorem 2.7 then says an organism must be complex. The implication on the concept of artificial life is this:

Theorem 2.8 *A living system must have noncomputable models.*

All the Rosen theorems have been mathematically proven. Counterexamples, therefore, cannot exist. For a detailed exposition of the underlying logic, the reader is encouraged to consult [2].

Note that Rosen's conclusion is *not* that artificial life is impossible. It is, rather, that life is *not computable*: however one models life, natural or artificial, one cannot succeed by computation alone. Life is not definable by an algorithm. There is, indeed, practical verification from computer science that attempts in implementation of a hierarchical closed loop leads to deadlock, and is hence forbidden in systems programming [6].

3 Set Theory

Let **N** be the collection of all natural systems. By Rosen's Definition 2.1, the set of all mechanisms is

$$\mathbf{M} = \{ N \in \mathbf{N} : \text{all models of N are simulable} \}.$$

Let me also define

$$\mathbf{Q} = \{ N \in \mathbf{N} : \text{in the category } \boldsymbol{C}(N) \text{ of models of } N \text{ the collections of analytic models and synthetic models coincide} \}.$$

and

$$\mathbf{R} = \{ N \in \mathbf{N} : \text{every property of } N \text{ is fractionable} \}.$$

Then Theorem 2.2 says that $\mathbf{M} \subset \mathbf{Q}$, while Theorem 2.3 says that $\mathbf{M} \subset \mathbf{R}$.

It is important to note that the five Rosen properties for a mechanism N are *necessary* properties. If N is a mechanism, then N necessarily has each one of these properties. Rosen only needed the necessity in these statements to establish the subsequent theorems. The five conclusions do *not* say the converse that if N has any one of these properties, then it is sufficient to guarantee that N is a mechanism. In particular, note that while $\mathbf{M} \subset \mathbf{Q}$, we in fact have $\mathbf{M} \neq \mathbf{Q}$.

Rosen himself fully realizes that **M** is a *proper* subset of **Q**. On p.186 of [4] he wrote "EVERY MODEL SIMULABLE IMPLIES ANALYTIC=SYNTHETIC and, to a sufficiently large extent, conversely." This says **M** is "almost" all of **Q**, but it is possible to find counterexamples of natural systems in **Q** but not in **M**.

The fatal error in the Chu-Ho paper [1] is that they misrepresent Rosen's definition of mechanism:

> "Rosen defines mechanisms as the class of systems of which all analytic models and synthetic models are equivalent" (first sentence of Section 2.4 of [1])

and then proceed to define mechanism erroneously:

> "we will give an alternative, yet essentially equivalent definition of mechanism: A system is a ***mechanism*** if all its analytic models are equivalent to synthetic models" (second paragraph in Section 3 of [1])

In other words, the set of mechanisms in the Chu-Ho definition is **Q**, not **M**.

The main argument in the Chu-Ho supposed demonstration of "why Rosen's central proof is wrong" appears to be their construction of a system with equivalent analytic and synthetic models (which they erroneously identified as a mechanism) but is nevertheless not fractionable. Their contention is that this provides a counterexample to Theorem 2.3, and therefore by gross generalization all of Rosen's results are suspect. They may or may not have an example of an element in **Q** but not in **R**, but that is irrelevant: Theorem 2.3 says $\mathbf{M} \subset \mathbf{R}$, *not* $\mathbf{Q} \subset \mathbf{R}$. From wrong definitions arise nonsensical conclusions.

Let me elaborate on what I mean by "wrong definition". Chu and Ho may of course define "mechanism" any way they please. After all, as Humpty Dumpty said to Alice: "When *I* use a word, it means just what I choose it to mean — neither more nor less." But from a non-Rosen definition of mechanism one cannot expect a Rosen property of mechanism to automatically follow. I again urge the reader to consult [2] to understand why the search of counterexamples to the Rosen theorems is futile.

4 (M,R)-Systems

Rosen devised a class of relational models of organisms called (M,R)-systems. He has discussed them on numerous occasions, including Section 10C of [4] and Chapter 17 of [5]. I have written on their noncomputability and realizations in two recent papers [2,3]. The reader may refer to any or all of the above for their details, which need not be repeated here.

For our present purpose, we only need to know that an (M,R)-system is an example of a formal system that has an impredicative loop of inferential entailment, hence nonsimulable. Chu and Ho attempt a discussion of (M,R)-systems in Section 4 of [1], but get it wrong as well.

It is telling that they claim "Wolkenhauer [7] offers a very clear discussion" on (M,R)-systems. But unfortunately Wolkenhauer does not correctly understand the concept of the replication map in (M,R)-systems. Chu and Ho simply quote Wolkenhauer, and therefore their replication map has the identical error. I shall use the Chu-Ho symbolism in my explanation in the following, although it is non-standard and awkward.

There are three mappings in an (M,R)-system on three hierarchical levels, and they entail one another in a cyclic permutation. They are

metabolism	$\mathtt{f}: A \to B$
repair	$F: B \to \mathtt{f}$
replication	$B: \mathtt{f} \to F$

The cyclic entailment pattern when we combines these three maps $\{\mathtt{f}, F, B\}$ is the closed hierarchical loop of inferential entailment in the (M,R)-system. Note that replication is $B: \mathtt{f} \to F$, not $\mathtt{f}: B \to F$ as claimed by Chu-Ho [1] and Wolkenhauer [7]. The correct relational diagram in graph-theoretic form is *not* Figure 7 of [1], but the following (again using the Chu-Ho symbolism):

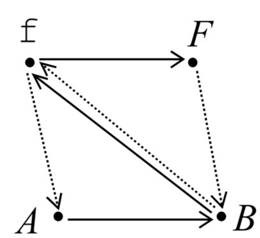

Figure 1. The correct entailment pattern of an (M,R)-system.

5 Artificial Life

Let me reiterate the fact that Rosen did *not* say that artificial life is impossible, only that life is *noncomputable*. Indeed, the subtitle of [4] is "A comprehensive enquiry into the nature, origin, and fabrication of life", with a positive elaboration on the last topic. Artificial life does not have to be limited to what a computing machine can do algorithmically. The first step is to admit that not everything is computable, i.e. throw away the Cartesian and Newtonian machine metaphor. One must loosen the mechanist constraints and assert the existence of natural systems with nonsimulable models.

Artificial life is not simply a *simulation* of life; it needs to be a *model* of life. The difference between a simulation and a model is that in the latter the morphism is mapped along with the domain and codomain, making the modeling relation a *natural transformation* in category-theoretic terms. (See Chapter 7 of [4] for details.) Artificial life must have entailment patterns that are congruent with the entailment patterns of living systems. These entailment patterns contain impredicative loops within themselves, and are beyond finite syntactic computation. A thorough discussion on the fabrication of life is found in Chapter 17 of [5], entitled "What does it take to make an organism?" Let me close by quoting a paragraph from it.

> On these grounds, we can see that the fabrication of something (e.g., an organism) is a vastly different thing than the simulation of its behaviors. The pursuit of the latter represents the ancient tradition that used to be called biomimesis, the imitation of life. The idea was that by serially endowing a machine with more and more of the simulacra of life, we would cross a threshold beyond which the machine would *become* an organism. The same reasoning is embodied in the artificial intelligence of today, and it is articulated in Turing's Test. This activity is a sophisticated kind of curve-fitting, akin to the assertion that since a given curve can be *approximated* by a polynomial, it must *be* a polynomial.

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